On a possible EPR experiment with $B_d^0 \overline{B_d^0}$ pairs

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Abstract. A very general local realistic theory of single B_d^0 mesons and of correlated $B_d^0 \overline{B_d^0}$ pairs is formulated. If these pairs are produced in $\Upsilon(4S)$ decay, the local realistic asymmetry for observing pairs with like and unlike flavour at different proper times remarkably differs from the quantum mechanical prediction. Asymmetric B factories provide a powerful tool for the study of the EPR problem since the relative detection experiments can be shown to be capable of a time-dependent measurement precise enough to discriminate between local realism and quantum theory.

1 Introduction

1.1 Historical view

The strange nature of quantum correlation between separated systems, pointed out for the first time by Einstein, Podolsky and Rosen (EPR) [1], has stimulated a lively debate. The incompatibility between quantum mechanics (QM) and local realism (LR) at the empirical level became evident with the 1965 work of Bell [2], showing that a wide class of local hidden variable models satisfies an inequality which is violated by QM. In 1969 Clauser, Holt, Shimony and Horne (CHSH) stressed that Bell's inequality could be experimentally checked with atomic cascade photon pairs emitted by single atoms [3]. Written in terms of detection probabilities, Bell's inequality becomes a double limitation on

$$\Gamma \equiv \omega(a,b) - \omega(a,b') + \omega(a',b) + \omega(a',b'), \quad (1)$$

and reads

$$-1 + \omega_1(a') + \omega_2(b) \leq \Gamma \leq \omega_1(a') + \omega_2(b), \quad (2)$$

where $\omega(x,y)$ is the joint probability that both photons are detected by two photomultipliers placed on their paths after crossing polarisers with axes x and y ($x = a, a'$ refers to the first photon, $y = b, b'$ refers to the second one); $\omega_1(a')$ $[\omega_2(b)]$ is the probability that the first [second] photon is transmitted by the polariser with axes a' [b] and detected. Inequalities (2) were not violated by the quantum mechanical predictions for experiments on atomic cascade photon pairs, but CHSH pointed out that meaningful experiments were possible, even with the available low efficiency photon counters, if the following *additional assumption* was made:

AA. Given that a pair of photons emerges from two regions of space where two polarisers can be located, the probability of their joint detection by two photomultipliers is independent of the presence and orientation of the polarisers. This assumption allowed CHSH to deduce a different inequality:

$$-1 + \omega(a',\infty) + \omega(\infty,b) \leq \Gamma \leq \omega(a',\infty) + \omega(\infty,b), \quad (3)$$

where $\omega(a',\infty)$ $[\omega(\infty,b)]$ is the joint probability that both photons are detected when the polariser is not present on the trajectory of the second [first] one, and Γ is again given by (1). The shift from (2) to (3) was crucial in producing a disagreement with QM. In an experiment in 1972 by Freedman and Clauser [4] the inequality (3) was violated, (2) was not (actually they experimentally studied a particular form of (3) – which is known as Freedman's inequality – to which of course our slightly more general argument applies fully). In 1974 Clauser and Horne [5] proposed a different additional assumption and built an explicit model of LR which reproduced within errors the results of the FC experiment. They could write: "We thereby prove that the Freedman–Clauser results constitute a refutation of only those OLT which satisfy our (or some similar) supplementary assumption", where OLT stands for "objective local theories", a notion equivalent to that of LR. In the 1981 Orsay experiments [6] the inequalities (2) and (3) respectively become

$$-0.845 \leq \Gamma \leq 0.155 \quad (4)$$

and

$$0.000 \leq \Gamma \leq 0.015, \quad (5)$$

when the lower and upper limits of (2) and (3) are calculated numerically for the actual experiment. The measured

value of Γ was found to agree with the prediction of QM and was slightly above 0.015, but well below 0.155. Therefore, inequality (4) was not violated, only (5) was. The meaning of the Orsay experiment is therefore totally dependent on the meaning of the inequality (3). As Clauser and Shimony noted [7], it is noteworthy that there exists an important hidden variable theory – the semiclassical radiation theory – which correctly predicts a large body of atomic physics data, but which violates the CHSH assumption AA. Moreover we stress that such an assumption is contrary to the fundamentals of the local realistic theories: emergence from two polarisers would in all cases imply a selection of the two-photon hidden variables, but these variables could well be those that determine the photomultiplier discharge! In 1983 Marshall et al. [8] showed that a consistently local and realistic picture of atomic cascades could fit the Orsay data as closely as the quantum mechanical model and identified in the low efficiency of photodetectors the reason for the compatibility between QM and LR in the case of all performed EPR experiments. During the eighties several papers [9] continued to stress the inequivalence of the inequalities (2) and (3), until in 1990 Lepore and Selleri [10] gave a general derivation of the two types of Bell's inequalities, those deduced from LR alone and those obtained with the essential help of an additional assumption (like AA above). These authors proposed to call the two types of inequalities "weak" and "strong" respectively. Thus (2) and (4) are weak, while (3) and (5) are strong: clearly (5) gives much stronger restrictions on Γ than (4). A recent general review of the strong and weak inequalities has been given in [11]. In spite of these unequivocal developments there still are papers which conclude that LR is experimentally violated without examining carefully the underlying assumptions. An example of how the experimental comparison between the existing quantum theory and LR should not be done is offered by a paper of the Rochester group [12] entitled *Experimental demonstration of the violation of local realism without Bell inequalities*. The physicists of this group compared their data with some consequences of LR obtained for ideal experiments [13]. In the abstract of the paper it is stated: "We report on a two-photon coincidence experiment that demonstrates the violation of local realism, as defined by Einstein, Podolsky and Rosen, by about 45 standard deviations [. . .]". If such a statement were correct the long debate on LR and the EPR paradox would finally have come to an end. However the analysis of the Rochester group appears immediately in a different light if one pays attention to an assumption made in the introduction of the paper [12]: "As the photodetectors used in the experiment are not 100% efficient [. . .] it is necessary to assume that photon absorptions are completely random. With that (fair sampling) assumption [. . .] we show that local realism is violated in two-photon coincidence experiments". Actually the fair sampling assumption (just like AA above) gives rise to a very important mutilation of LR; this is known at least since 1983 when that assumption was proposed in Aspect's thesis [14]. A first answer to the Rochester group was given by Garuc-

cio [15] who could show that a crucial parameter η_0 (ratio of sums of quantum mechanical probabilities) must have a value not less than 0.82 in order to make a meaningful comparison between QM and LR. The actual value for the Rochester experiment was 0.10 and Garuccio rightly concluded [15]: "[. . .] this proves that the experiment is not able to distinguish quantum mechanics and Einstein locality without supplementary assumptions". Here "Einstein locality" is practically synonymous to LR. Because of the low detection efficiency the same considerations apply to the recent experiment by Zeilinger [16]. A second answer was given by Cabello and Santos [17] who built a local hidden variable model whose predictions were in numerical agreement with the photon coincidence numbers observed in the Rochester experiment and collected in Table 1 of [12]. According to the Rochester group those numbers implied a violation of LR, but Cabello and Santos wrote: "We maintain that the experiment refutes only a restricted family of local hidden variable theories containing additional assumptions". The comparison between LR and QM at the beginning of the year 2000 is as open as it always was. Hopes of a final resolution of the EPR paradox lie in the study of K^0–$\overline{K^0}$ pairs produced at ϕ factories and of B^0–$\overline{B^0}$ pairs produced at B factories, because particle physics detectors are much more efficient than those for optical photons and the distinction between "strong" and "weak" formulations of LR becomes much less important, both conceptually and numerically.

1.2 EPR tests in particle physics

The behaviour of a pair of neutral pseudo-scalar mesons (e.g. $K^0\overline{K^0}$ or $B^0\overline{B^0}$), anti-correlated in flavour if produced by the decay of a $J^{PC} = 1^{--}$ state (e.g. the Φ or the $\Upsilon(4S)$ resonances) seems even more puzzling than the behaviour of spin correlated pairs of stable particles (photons or electrons). A critical discussion of the EPR paradox for $K^0\overline{K^0}$ pairs and of the earlier attempts to check Bell's inequality in kaon Φ decays was made by Ghirardi, Grassi and Weber [18]. The main argument was that Bell's inequality, written in terms of four different times of flight of the kaons, is not violated by the quantum mechanical two-time joint probability for correlated strangeness, due to the specific values of kaon masses and decay widths. It could easily be shown in the same way that an analogous argument holds for B_d mesons. Recently further Bell-type tests involving new Bell-like inequalities for correlated neutral meson–anti-meson pairs have been proposed and discussed [19]. However, some of them would probe only a restricted class of local realistic theories whereas others avoid this difficulty but require experimental set-ups not available in the near future [11]. But Bell's inequality is only one of the many consequences of LR: EPR correlations can provide tests sensitive to possible deviations from QM. Meaningful tests of LR, not of the Bell-type, have been proposed for the K^0–$\overline{K^0}$ system [20] [21]. In the present paper, after developing a very general local realistic theory for the B_d^0–$\overline{B_d^0}$ system (following

the ideas in [20] and [21]), a test to discriminate between QM and LR is proposed and its feasibility with an asymmetric B factory is discussed. Only a few works on EPR tests refer to the B_d^0–$\overline{B_d^0}$ system [22] and use a *decoherence parameter* ζ (such that the QM interference term is multiplied by a factor $(1-\zeta)$) to measure deviations from QM. Combining already published data (from CLEO and Argus and from LEP experiments) they deduce a clear indication in favour of QM. However, this is an expected conclusion taking into account that falsification of $\zeta = 1$ means simply a falsification of the spontaneous factorization hypothesis (SFH). Indeed SFH was already falsified with gamma ray pairs from e^-–e^+ annihilations [23], with atomic photon pairs [24] and recently with kaons pairs produced in p–$\bar{p}$ annihilations [25]. On the other hand, it is possible to reproduce within the local realistic approach all non-paradoxical predictions of QM, including anti-correlations in strangeness and CP (or beauty and mass), which are absent in SFH; thus SFH violation is also predicted by any reasonable approach based on LR. In the present paper we will deduce from LR physical consequences in a completely general way – so general that if they are found incorrect in a high efficiency experiment on correlated neutral B_d meson pairs, this will imply the death of LR. Such an event would be entirely new. This work is organized as follows. In the Sect. 2 the B_d meson pair correlation and the quantum mechanical formalism are reviewed. In the Sect. 3 a local realistic theory for the B_d^0–$\overline{B_d^0}$ system is accurately developed and its predictions, different from those of QM, are discussed. In Sect. 4 we stress that the asymmetric collider allows one to perform time-dependent measurements and then discuss how our EPR test can be performed using techniques and methods peculiar to lifetime and mixing measurement.

2 B meson pair correlation

Let us briefly review the quantum mechanical basic formalism for the EPR correlated $B^0 \overline{B^0}$ pairs that can be created as decay products of the $\Upsilon(4S)$ resonance. More precisely $B_d^0 \overline{B_d^0}$ pairs are produced since $\Upsilon(4S)$ is not heavy enough to decay into $B_s^0 \overline{B_s^0}$ pairs. From now on the index d will be dropped to simplify notation: with B^0 a B_d^0 will be intended. In the quark model the resonance $\Upsilon(4S)$ is a $b\bar{b}$ state with quantum numbers $J^{PC} = 1^{--}$. In the strong decay $\Upsilon(4S) \to B^0 \overline{B^0}$ the created pair inherits the $\Upsilon(4S)$ quantum numbers. Since B mesons are spinless, $J^P = 1^-$ implies that the $B^0 \overline{B^0}$ pair is in a p-wave total angular momentum state. $C = -1$ implies that the flavour part of the pair wave function is anti-symmetric. Therefore immediately after the decay (namely at $t = 0$) the $B^0 \overline{B^0}$ state vector is given, in the $\Upsilon(4S)$ frame, by

$$|\psi(0,0)\rangle = \frac{1}{\sqrt{2}}\{|B^0(\vec{p})\rangle|\overline{B^0}(-\vec{p})\rangle$$
$$- |\overline{B^0}(\vec{p})\rangle|B^0(-\vec{p})\rangle\}$$
$$\equiv \frac{1}{\sqrt{2}}\{|B^0\rangle_l|\overline{B^0}\rangle_r - |\overline{B^0}\rangle_l|B^0\rangle_r\}, \quad (6)$$

where l (r) denote the B meson motion directions "left" ("right") and $|B^0\rangle$, $|\overline{B^0}\rangle$ are beauty eigenstates ($B^0 = \bar{b}d$, $\overline{B^0} = b\bar{d}$). Possible tiny CP violation will be neglected throughout this paper since it could not appreciably modify the large difference between LR and QM predictions that will be found. Thus the mass eigenstates $|B_H^0\rangle$, $|B_L^0\rangle$ (H for "heavy" and L for "light" such that $\Delta m = m_H - m_L > 0$) can be identified with CP eigenstates:

$$|B_H^0\rangle = \frac{1}{\sqrt{2}}\{|B^0\rangle + |\overline{B^0}\rangle\}, \quad |B_L^0\rangle = \frac{1}{\sqrt{2}}\{|B^0\rangle - |\overline{B^0}\rangle\}, \quad (7)$$

and the state vector (6) can also be expressed as

$$|\psi(0,0)\rangle = \frac{1}{\sqrt{2}}\{|B_H^0\rangle_l|B_L^0\rangle_r - |B_L^0\rangle_l|B_H^0\rangle_r\}. \quad (8)$$

The time evolution of the complex mass eigenstates is given by

$$|B_H^0(t)\rangle = |B_H^0\rangle e^{-\alpha_H t}, \quad |B_L^0(t)\rangle = |B_L^0\rangle e^{-\alpha_L t}, \quad (9)$$

where t is the particle proper time and (with $\hbar = c = 1$)

$$\alpha_H = \frac{1}{2}\Gamma + i m_H, \quad \alpha_L = \frac{1}{2}\Gamma + i m_L \quad (10)$$

(the two neutral B mesons are expected to have a negligible difference in lifetime). The time evolution operator for state (6) is the product of the time evolution operators for single B mesons states, so that, at proper times t_l and t_r, the time evolved state for the $B^0 \overline{B^0}$ pair can be written as

$$|\psi(t_l, t_r)\rangle = \frac{1}{\sqrt{2}}\{|B_H^0\rangle_l|B_L^0\rangle_r e^{-\alpha_H t_l - \alpha_L t_r}$$
$$- |B_L^0\rangle_l|B_H^0\rangle_r e^{-\alpha_L t_l - \alpha_H t_r}\}. \quad (11)$$

The different exponentials in (11) generate $B^0 B^0$ and $\overline{B^0}\overline{B^0}$ components. Indeed the time evolution of B^0, $\overline{B^0}$ is governed by a weak interaction that does not conserve flavour and thus allows B^0–$\overline{B^0}$ oscillations to take place between the flavour eigenstates (B^0–$\overline{B^0}$ *mixing*). The probabilities of $B^0 \overline{B^0}$, $B^0 B^0$ and $\overline{B^0}\overline{B^0}$ observations are given (using 11) and (7)) by

$$P^{QM}[B^0(t_l); \overline{B^0}(t_r)]$$
$$= \frac{1}{4}E(t_r + t_l)[1 + \cos(\Delta m(t_r - t_l))]$$
$$= P^{QM}[\overline{B^0}(t_l); B^0(t_r)], \quad (12)$$

$$P^{QM}[B^0(t_l); B^0(t_r)]$$
$$= \frac{1}{4}E(t_r + t_l)[1 - \cos(\Delta m(t_r - t_l))]$$
$$= P^{QM}[\overline{B^0}(t_l); \overline{B^0}(t_r)], \quad (13)$$

where $E(t) \equiv e^{-\Gamma t}$. For $t_l = t_r$ (13) vanishes and the probability of having like flavours is zero. This flavour anti-correlation means that the two neutral B mesons evolve

in phase so that at equal proper times, until one of them decays, there are always a B^0 and a $\overline{B^0}$ present. Let us consider the following flavour asymmetry

$$A(t_l, t_r) = \frac{P[B^0(t_l); \overline{B^0}(t_r)] - P[\overline{B^0}(t_l); \overline{B^0}(t_r)]}{P[B^0(t_l); \overline{B^0}(t_r)] + P[\overline{B^0}(t_l); \overline{B^0}(t_r)]}, \quad (14)$$

that will be our fundamental parameter for the comparison between quantum theory and the local realistic predictions. From the experimental point of view the advantage of considering ratios of probabilities or numbers of events is that systematic errors tend to cancel. In QM the asymmetry (14) is predicted to be a very simple function of $t_r - t_l$ only (by (12) and (13)):

$$A^{\mathrm{QM}}(t_l, t_r) = \cos(\Delta m(t_r - t_l)) = \cos\left(x\left(\frac{t_r - t_l}{\tau}\right)\right), \quad (15)$$

where [26]

$$x = \frac{\Delta m}{\Gamma} = 0.723 \pm 0.032. \quad (16)$$

We will show that the asymmetry predicted by LR is necessarily quite different from (15).

3 B meson pair correlations within LR

3.1 Two-time probabilities

The EPR paradox arises from the incompatibility at the empirical level between the predictions of quantum theory and local realism. The latter can be expressed by the following three assumptions:

(I)   If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity (*EPR reality criterion*).

(II)   If two physical systems (e.g. our two neutral B mesons) are separated by a large distance, an element of reality belonging to one of them cannot have been created by a measurement performed on the other one (*separability*).

(III) If at a given time t a physical system has an element of reality, the latter cannot be created by measurements performed on the same system at time $t' > t$ (*no retroactive causality*).

Local realism can be applied to the B^0 meson pairs described quantum mechanically by the state vector (11), by considering only those predictions of (11) to which the EPR reality criterion can be applied. These are *flavour* and *mass* anti-correlations. Obviously, if such anti-correlations were not found to exist experimentally, our conclusions could not be correct. If they are assumed to exist, one can say the following [20]:

(a) each B meson of every pair has an associated element of reality λ_1 which determines a well defined value of mass ($\lambda_1 = +1, -1$ corresponds to $m_{\mathrm{H}}, m_{\mathrm{L}}$, respectively);

(b) each B meson of every pair has an associated element of reality λ_2 which determines a well defined value of flavour ($\lambda_2 = +1, -1$ corresponds to $b = +1, -1$, respectively).

Furthermore b is not a stable property: it has sudden jumps, from $+1$ to -1 and vice versa, that are simultaneous for the two mesons of every pair but happen at random times in a statistical ensemble of many pairs. Notice that the application of local realism to the physical situation described by (11) has brought us, at least formally, outside quantum theory: no quantum mechanical state vector exists, in fact, which can describe a B meson as having simultaneously well-defined mass and flavour values. This is of course the standard approach of all realistic interpretations of quantum phenomena. Following the treatment in [20] and [21] relative to K mesons and taking into account the analogy (strangeness, CP) $\leftrightarrow$ (beauty, mass), four B meson basic states, characterised by beauty and mass both definite, can be introduced:

$$\begin{aligned} B_1 &= B_{\mathrm{H}}: \quad \text{state with } b = +1 \text{ and } m = m_{\mathrm{H}}, \\ B_2 &= \overline{B_{\mathrm{H}}}: \quad \text{state with } b = -1 \text{ and } m = m_{\mathrm{H}}, \\ B_3 &= B_{\mathrm{L}}: \quad \text{state with } b = +1 \text{ and } m = m_{\mathrm{L}}, \\ B_4 &= \overline{B_{\mathrm{L}}}: \quad \text{state with } b = -1 \text{ and } m = m_{\mathrm{L}}. \end{aligned} \quad (17)$$

The probabilities of observing the ith state at proper time t conditional on an initial jth state at proper time zero are $p_{ij}(t|0) = $ probability of B_i at time t given B_j at time 0 ($i, j = 1, 2, 3, 4$) for the left going (l) meson, and $q_{ij}(t|0) = $ probability of B_i at time t given B_j at time 0 ($i, j = 1, 2, 3, 4$) for the right going (r) meson. The symbols p and q will be used for all the probabilities of the left and right going meson, respectively. It can be shown that all the physical conditions imposed by QM to single B mesons are satisfied by the probabilities of LR which can be collected in a "probability matrix"

$$M = \quad (18)$$

$$\begin{pmatrix} E(t)Q_+(t) & E(t)Q_-(t) & 0 & 0 \\ E(t)Q_-(t) & E(t)Q_+(t) & 0 & 0 \\ 0 & 0 & E(t)Q_+(t) & E(t)Q_-(t) \\ 0 & 0 & E(t)Q_-(t) & E(t)Q_+(t) \end{pmatrix},$$

where $p_{11}(t|0) = q_{11}(t|0) = E(t)Q_+(t)$, $p_{12}(t|0) = q_{12}(t|0) = E(t)Q_-(t)$, etc., and

$$Q_{\pm}(t) \equiv \frac{1}{2}[1 \pm \cos(\Delta m t)], \quad (19)$$

so that $Q_+(t) + Q_-(t) = 1$. The proof of (18) is strictly analogous to that for neutral kaons published in [20]. It has been shown that this set of probabilities is unique within the local realistic approach [11].

3.2 Three time probabilities

The physical situation described in QM by the wave function (6) corresponds to the initial LR probabilities

$$q_1(0) = q_2(0) = q_3(0) = q_4(0) = \frac{1}{4}, \quad (20)$$

where q_i is the probability of the state B_i ($i = 1, 2, 3, 4$). Our present task is to find the maximum and minimum values allowed by LR for the asymmetry parameter A, for $t_r > t_l$. Two time probabilities can be written by means of probabilities with three indices by using Bayes' formula [27]:

$$q_{11}(t_r, t_l) = q_{111}(t_r, t_l|0)q_1(0) + q_{112}(t_r, t_l|0)q_2(0),$$
$$q_{21}(t_r, t_l) = q_{211}(t_r, t_l|0)q_1(0) + q_{212}(t_r, t_l|0)q_2(0), \quad (21)$$

where $q_{ijk}(t_r, t_l|0)$ is the probability of having B_i at time t_r and B_j at time t_l given a B_k at time 0 ($i, j, k = 1, 2, 3, 4$). To simplify the notation, let us set $E_l \equiv E(t_l)$, $Q_\pm^l \equiv Q_\pm(t_l)$ and $E_r \equiv E(t_r)$, $Q_\pm^r \equiv Q_\pm(t_r)$. One must have

$$q_{111}(t_r, t_l|0) + q_{121}(t_r, t_l|0) = E_r Q_+^r, \quad (22)$$

$$q_{211}(t_r, t_l|0) + q_{221}(t_r, t_l|0) = E_r Q_-^r, \quad (23)$$

because the left hand sides are the probabilities of B_i at time t_r ($i = 1, 2$) summed over all possible states at time t_l, given a B_1 at time 0. Therefore the left hand sides of (22) and (23) must equal $q_{11}(t_r|0)$ and $q_{21}(t_r|0)$, which are given by (18). To (22) and (23) we can add

$$q_{111}(t_r, t_l|0) + q_{211}(t_r, t_l|0) = E_{r-l} q_{11}(t_l|0), \quad (24)$$

$$q_{121}(t_r, t_l|0) + q_{221}(t_r, t_l|0) = E_{r-l} q_{21}(t_l|0), \quad (25)$$

where $E_{r-l} \equiv E(t_r - t_l)$. The left hand side of (24) is the probability that the meson is either $B_1(t_r)$ or $B_2(t_r)$ (then that is undecayed at t_r) or that it is $B_1(t_l)$, given that it was $B_1(0)$. The meaning of (25) is similar. The probabilities on the right hand sides of (24) and (25) are again given by (18). Therefore, (24) and (25) can be written as

$$q_{111}(t_r, t_l|0) + q_{211}(t_r, t_l|0) = E_r Q_+^l, \quad (26)$$

$$q_{121}(t_r, t_l|0) + q_{221}(t_r, t_l|0) = E_r Q_-^l, \quad (27)$$

because of $E_r = E_{r-l} E_l$. Four unknown probabilities appear in (22), (23) and (26), (27). These equations are however not independent because the sum of (22) and (23) equals the sum of (26) and (27) (remember that $Q_+ + Q_- = 1$). One can write three probabilities in terms of q_{111}:

$$\begin{cases} q_{121} = E_r Q_+^r - q_{111}, \\ q_{211} = E_r Q_+^l - q_{111}, \\ q_{221} = E_r(Q_-^l - Q_+^r) + q_{111}, \end{cases} \quad (28)$$

where the arguments $(t_r, t_l|0)$ of the three time probabilities have been omitted; this will be done systematically in the following. Obvious upper and lower limits for q_{111} are

$$0 \le q_{111} \le E_r Q_+^r, E_r Q_+^l, \quad (29)$$

as follows from (22) and (26). Analogous limits can be obtained for the other three probabilities which, using (28), can be translated into further limitations for q_{111}. The overall result is

$$p \le q_{111} \le P, \quad (30)$$

where

$$p = E_r \mathrm{Max}\{0; (Q_+^r - Q_-^l)\},$$
$$P = E_r \mathrm{Min}\{Q_+^l; Q_+^r\}. \quad (31)$$

When different limitations are given it is useful to adopt the most stringent ones. For this reason in (31) the maximum of the minima and the minimum of the maxima are considered. Which one of the two terms within curly brackets has to be chosen depends on t_l and t_r; for instance $Q_+^r - Q_-^l$ can be positive or negative depending on the considered times. Equations similar to (22), (23) and (26), (27) can be written for an initial B_2. They are

$$q_{222} + q_{212} = E_r Q_+^r, \quad (32)$$

$$q_{122} + q_{112} = E_r Q_-^r \quad (33)$$

and

$$q_{222} + q_{122} = E_r Q_+^l, \quad (34)$$

$$q_{212} + q_{112} = E_r Q_-^l. \quad (35)$$

Notice that if the indices 1 and 2 are systematically interchanged, (22), (23) and (26), (27) become (32), (33) and (34), (35) respectively. Conclusions similar to (28)–(30) must then hold for these new probabilities, by applying the same interchange. One has

$$\begin{cases} q_{212} = E_r Q_+^r - q_{222}, \\ q_{122} = E_r Q_+^l - q_{222}, \\ q_{112} = E_r(Q_-^l - Q_+^r) + q_{222} \end{cases} \quad (36)$$

and

$$p \le q_{222} \le P, \quad (37)$$

with p and P again given by (31). Remembering (21) and (20), and using (28) and (36), one gets

$$q_{11}(t_r, t_l) = \frac{1}{4}(q_{111} + q_{112})$$
$$= \frac{1}{4}[E_r(Q_-^l - Q_+^r) + q_{111} + q_{222}], \quad (38)$$

$$q_{21}(t_r, t_l) = \frac{1}{4}(q_{211} + q_{212})$$
$$= \frac{1}{4}[E_r(Q_+^l + Q_+^r) - q_{111} - q_{222}]. \quad (39)$$

Similar reasonings can be made for $B_3(0)$ and $B_4(0)$. The results are very similar to the previous case and one obtains

$$q_{33}(t_r, t_l) = \frac{1}{4}(q_{333} + q_{334})$$
$$= \frac{1}{4}[E_r(Q_-^l - Q_+^r) + q_{333} + q_{444}], \quad (40)$$

$$q_{43}(t_r, t_l) = \frac{1}{4}(q_{433} + q_{434})$$
$$= \frac{1}{4}[E_r(Q_+^l + Q_+^r) - q_{333} - q_{444}], \quad (41)$$

with the limitations

$$p \le q_{333} \le P, \qquad p \le q_{444} \le P, \quad (42)$$

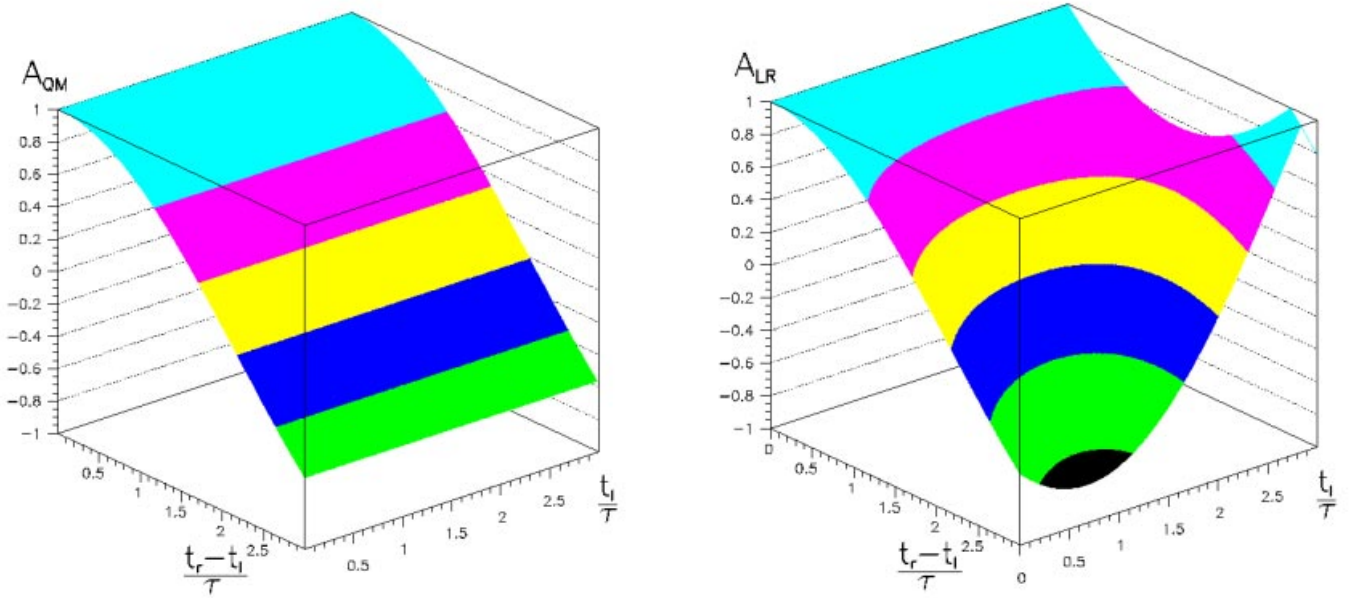with p and P again given by (31).

Fig. 1. Flavour asymmetry predicted by QM (left) and LR (right; maximum values)

3.3 B meson pair probabilities

Next we calculate the local realistic probability of having two B^0 mesons with $b = -1$ at proper times t_l and $t_r > t_l$. We must then consider the situations in which there is either B_2 or B_4 on the left at time t_l and on the right at time t_r. This probability is

$$P^{LR}[\overline{B^0}(t_l); \overline{B^0}(t_r)] = E_l q_{21}(t_r, t_l) + E_l q_{43}(t_r, t_l). \quad (43)$$

The first term in (43) is the probability that the first B^0 is undecayed at time t_l times the probability that the second B^0 is $B_2(t_r)$ and $B_1(t_l)$: the latter factor, due to anticorrelation, coincides with the probability that the second B^0 is $B_2(t_r)$ and the first one $B_4(t_l)$. Similar is the meaning of the second term in (43). We must also calculate the probability of having a B^0 with $b = +1$ at proper time t_l and a B^0 with $b = -1$ at proper time $t_r > t_l$. This is clearly given by

$$P^{\mathrm{LR}}[\overline{B^0}(t_l); B^0(t_r)] = E_l q_{33}(t_r, t_l) + E_l q_{11}(t_r, t_l). \quad (44)$$

From (43) and (44) it follows that

$$P^{\mathrm{LR}}[\overline{B^0}(t_l); B^0(t_r)] + P^{\mathrm{LR}}[\overline{B^0}(t_l); \overline{B^0}(t_r)]$$
$$= E_l[q_{21} + q_{11} + q_{43} + q_{33}]. \quad (45)$$

From (38)–(41) one also has

$$P^{\mathrm{LR}}[\overline{B^0}(t_l); B^0(t_r)] + P^{\mathrm{LR}}[\overline{B^0}(t_l); \overline{B^0}(t_r)] = \frac{1}{2} E_{r+l}. \quad (46)$$

Furthermore

$$P^{\mathrm{LR}}[\overline{B^0}(t_l); B^0(t_r)] - P^{\mathrm{LR}}[\overline{B^0}(t_l); \overline{B^0}(t_r)]$$
$$= E_l[q_{33} - q_{43}] + E_l[q_{11} - q_{21}]. \quad (47)$$

Using (38)–(41) and the limits (30), (37) and (42) it follows that

$$\{P^{\mathrm{LR}}[\overline{B^0}(t_l); B^0(t_r)] - P^{\mathrm{LR}}[\overline{B^0}(t_l); \overline{B^0}(t_r)]\}_{\max}$$
$$= \frac{1}{2} E_{r+l}(1 - 2 \mid Q_+^l - Q_+^r \mid), \quad (48)$$

and

$$\{P^{\mathrm{LR}}[\overline{B^0}(t_l); B^0(t_r)] - P^{\mathrm{LR}}[\overline{B^0}(t_l); \overline{B^0}(t_r)]\}_{\max}$$
$$= \frac{1}{2} E_{r+l}\{1 - 2\mathrm{Min}[(Q_+^l + Q_+^r); (Q_-^l - Q_-^r)]\}. \quad (49)$$

Thus, for LR, the asymmetry parameter (14) turns out to have maximum and minimum values given by

$$\begin{cases} A_{\max}^{\mathrm{LR}}(t_l; t_r) = 1 - 2 \mid Q_+^l - Q_+^r \mid, \\ A_{\min}^{\mathrm{LR}}(t_l; t_r) = 1 - 2\mathrm{Min}[(Q_+^l + Q_+^r); (Q_-^l + Q_-^r)], \end{cases} \quad (50)$$
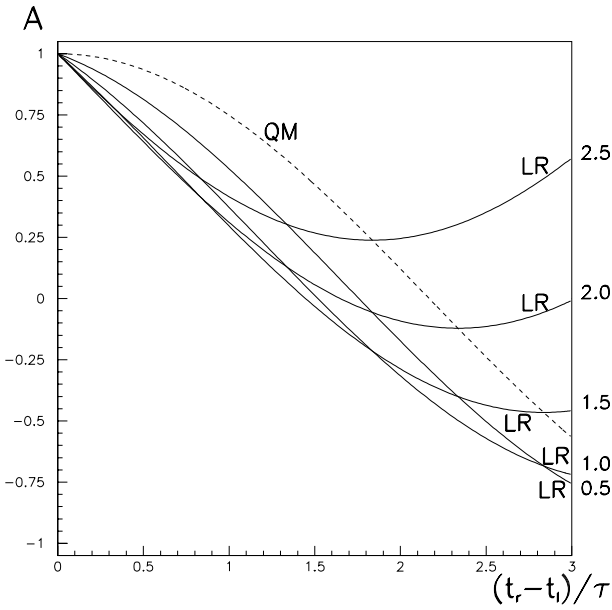
It is useful to write (50) again with the explicit dependence on $(t_r - t_l)/\tau \equiv \eta$ (with $\eta > 0$):

$$\begin{cases} A_{\max}^{\mathrm{LR}}(t_l; t_r) = 1 - \mid (1 - \cos(x\eta)) \cos(xt_l) \\ \qquad\qquad + \sin(x\eta) \sin(xt_l) \mid, \\ A_{\min}^{\mathrm{LR}}(t_l; t_r) = 1 \\ \qquad\qquad - \mathrm{Min}[2 + \Psi; 2 - \Psi], \end{cases} \quad (51)$$

where

$$\Psi = (1 + \cos(x\eta)) \cos(xt_l) - \sin(x\eta) \sin(xt_l). \quad (52)$$

The behaviour of (15) and (50) is shown in Fig. 1. It can be noticed that A^{QM} depends only on the difference of the two proper times and remains unchanged under the $t_l \leftrightarrow t_r$ exchange (thus showing a symmetric behaviour

Fig. 2. Flavour asymmetry predicted by QM and LR (maximum values) for some fixed t_l/τ values

with respect to $(t_r - t_l)/\tau$. Therefore, with $t_l > t_r$, the study of A^{QM} is reduced to the case $t_r > t_l$. On the other hand A^{LR} depends not only on this difference but also on one given absolute proper time, t^*. However, under the $t_l \leftrightarrow t_r$ exchange A^{LR} is symmetric as can easily be shown and as should be expected from the conventionality of the l and r tags. Thus the study of A^{LR} can be limited to the case $t_r \geq t_l$ establishing, conventionally, the dependence of A^{LR} on the shorter absolute time. To appreciate the difference between the prediction of QM and the maximum prediction of LR it is useful to represent the asymmetry parameter as a function of $(t_r - t_l)/\tau$ for some given values of t_l as in Fig. 2. The comparison of the whole LR prediction with the MQ one, for two representative values of t_l, is given in Fig. 3. It is interesting to notice that the difference between the predictions of QM and LR is higher in the B^0–$\overline{B^0}$ system than in the K^0–$\overline{K^0}$ system, after comparison with the results of [21]. However, from an experimental point of view, B mesons require measuring smaller decay times that is more difficult.

4 Experimental feasibility of the EPR test

The construction of the asymmetric B factories (high luminosity asymmetric e^+e^- colliders operating at the $\Upsilon(4S)$ resonance), i.e. the PEP-II storage ring instrumented with the BaBar detector at SLAC [28], and the KEK-B storage ring with the Belle detector at KEK [29], will provide a powerful tool to perform the EPR test just proposed. Indeed, as we are going to argue, this test can be performed with the experimental setups and software tools already being developed for CP violation, lifetime and

mixing measurements, and the precision required for these measurements is adequate.

Firstly we review some characteristics and experimental parameters concerning asymmetric B factories, then we suggest two methods to perform the EPR test and finally we provide a quantitative evaluation for the test feasibility.

4.1 Methods and performances at the asymmetric B factories

The asymmetry (14), as predicted by QM, depends on the difference of the two B meson proper decay times Δt. The asymmetric collider configuration and an accurate vertex determination make the determination of Δt possible. Indeed, the fundamental reason for developing asymmetric B factories is the need to measure small proper time intervals, whereas a precise vertex detector is necessary to measure a space dependence from which the time dependence can be deduced. Time measurements would not be feasible if the $\Upsilon(4S)$, having an energy just over the B_d meson pair production threshold, were produced at rest: the B mesons have very low momentum ($340 \, \text{MeV}/c$) in the center-of-mass frame and the paths between production and decays are too small ($30 \, \mu\text{m}$ on average) to be measured even by silicon vertex detectors. Therefore, proper decay time measurements require the asymmetric beam configuration that produces the $\Upsilon(4S)$ resonance in motion in the laboratory frame, thus allowing the expansion between the two decay positions. Neglecting B mesons motion in the $\Upsilon(4S)$ frame, the proper time difference is to a good approximation given by $\Delta t \simeq \Delta z/c\eta$ where Δz is the distance between the two B decay vertices measured in the laboratory, along the beams direction, and $\eta = \beta\gamma$ is the Lorentz boost factor due to the asymmetric beam energies. Its value is chosen as a good compromise between the Δz expansion and the necessity that the boosted decay products are not lost in the dead cone ahead avoiding efficiencies decrease: $\eta = 0.56$ at BaBar-PEP II [28], $\eta = 0.42$ at Belle-KEKB [29]. The applied boost expands the average separation of the two B vertices Δz to about $250 \, \mu\text{m}$. In this case a silicon vertex detector allows one to determine the separation of the two B decay vertices Δz. Both BaBar [28] and Belle [29] can provide a $\sigma_{\Delta z}$ of the order of $100 \, \mu\text{m}$ and in some cases better, depending on the detected decay channels. Among others three B physics measurements can be performed at asymmetric B factories: CP asymmetries, B_d mixing and the B_d lifetime. Both physical time-dependent asymmetries, that need to be measured by BaBar and Belle in order to establish CP violation and determine the mixing parameter Δm_B, depend on the difference Δt between the two B meson proper decay times. To measure CP asymmetries one looks for events where one neutral B meson decays into an hadronic CP eigenstate at the time t_{CP} while the other decays, at t_{tag}, to a semi-leptonic or hadronic tagging mode that acts as a b flavour identifier: $\Delta t = t_{CP} - t_{\text{tag}}$. On the other hand the mixing measurement can be based on double tagged dileptonic events. It must be pointed out that the
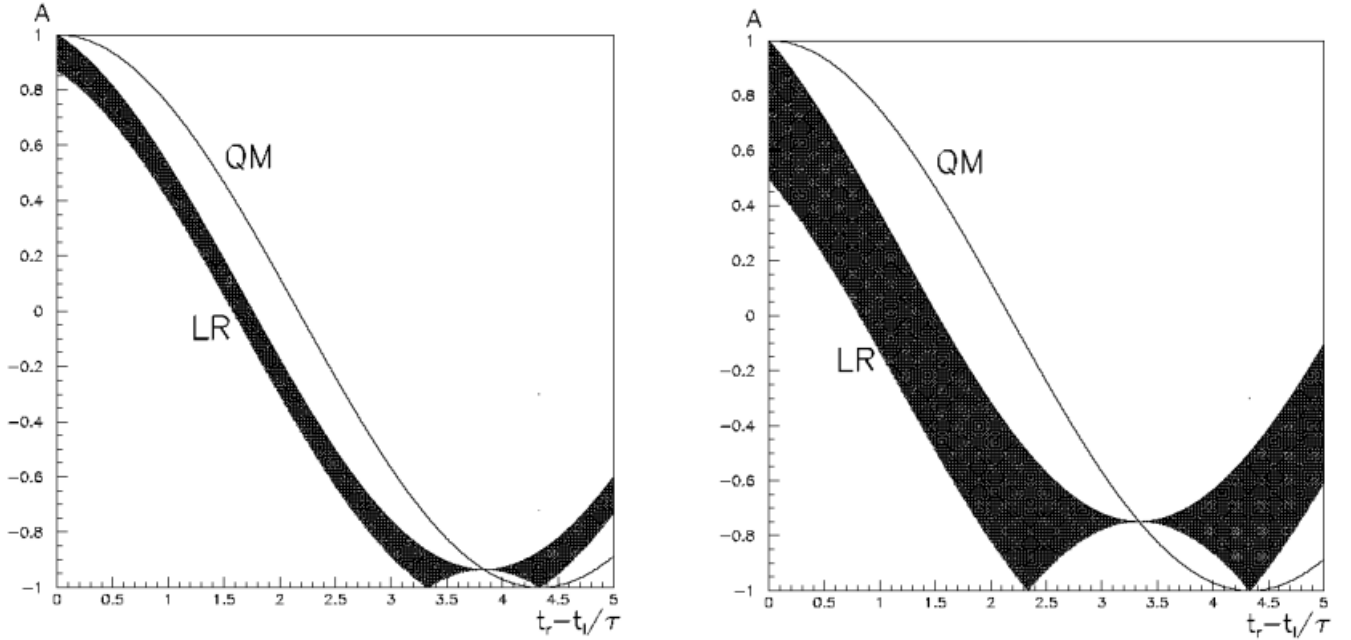
Fig. 3. Flavour asymmetry predicted by QM and LR for $t_1/\tau = 0.5$ (left) and 1.0 (right)

dilepton approach with a time-dependent asymmetry is different from the usual time-integrated dilepton methods developed at a symmetric e^+e^- collider at $\Upsilon(4S)$ (by the CLEO experiment) which allows only the measurement of $\chi_d = x_d^2/2(1 + x_d^2)$. Most lifetime measurement methods rely on the determination of both the secondary vertices and of the primary one, the $\Upsilon(4S)$ decay point; an exclusive reconstruction for at least one B meson is needed, while the other B can be partially reconstructed or only vertexed.

In general the resolution achievable in individual track reconstruction is limited by multiple scattering introduced primarily by the beam pipe. The resolution $\sigma_{\Delta z}$ (for example $\sigma_{\Delta z} = (\sigma_{z_{CP}}^2 + \sigma_{z_{tag}}^2)^{1/2}$) is primarily determined by the error in the tagged vertex position $\sigma_{z_{tag}}$ [28] [29]. Typical values are $\sigma_z \simeq 50\,\mu m$ for a fully reconstructed B meson and $\sigma_z \simeq 100\,\mu m$ for a partially reconstructed one. In clean hadronic channels and in lepton tagging a better resolution can be achieved in principle, thus together providing $\sigma_{\Delta z}$ better than $100\,\mu m$ for the CP analysis and $\sigma_{\Delta z} \simeq 110\,\mu m$ in the dilepton analysis. On the other hand non-gaussian tails with $\sigma \sim 200 \div 300\,\mu m$ for a significant fraction of the selected samples may partially spoil these resolutions. In conclusion, we assume that the asymmetric machine configuration and high performance in B vertexing will provide time-dependent measurements with $\sigma_{\Delta z}$ of the order of $100 \div 110\,\mu m$ so that $\sigma_{\Delta t} = \sigma_{\Delta z}/\beta\gamma c \sim 0.4\,\tau$.

4.2 EPR test performed by lifetime measurement method

The $\Upsilon(4S)$ decays almost exclusively to pairs of B mesons, that are neutral with a probability of about 50%. In such cases no ionizing particle comes out of the $\Upsilon(4S)$ decay point which is thus not directly measurable. This in turn does not permit one to record and use decay distances in a lifetime determination. However, lifetime measurements can be performed without knowing the B mesons production point, using only the Δz measurement. On the other hand a more sophisticated method of reconstructing the geometry and the kinematics of the events has been developed in BaBar in order to perform more direct lifetime measurements [28]. This method can suggest how to perform a measurement of the asymmetry (14) by measuring not only Δt but also one time of flight ($t^* = t_1$, Sect. 3.3) as needed by the local realistic time dependence. In this method the decay vertices of the fully reconstructed B^0, of the other B^0 and the $\Upsilon(4S)$ production point, considered to belong to the very flat ribbon-shaped beam spot ($\sigma_x \simeq 140\mu m$, $\sigma_y \simeq 6\,\mu m$, $\sigma_z \simeq 1\,cm$), are connected assuming the direction of their lines of flight from the kinematics. Thus a gain in measurement accuracy is provided by adding kinematical constraints to the standard geometrical fit of the decay vertices. To perform the EPR test the quantities $\sigma_{\Delta t}$, σ_{t^*} must be both small enough in order to allow for an effective time-dependent analysis. Qualitatively $\sigma_{\Delta z}$ and σ_{z-z_Υ} should be smaller than half the average separation between the vertices, and the typical value of $\sigma_{\Delta z} \sim 100\,\mu m$ ($\sigma_{\Delta t} \sim 0.4\tau$) could be adequate. This requirement could be satisfied with both the B mesons fully reconstructed and using the algorithm mentioned above. On the other hand, as the rate of fully

reconstructible B mesons is likely to be of the order of 1% or less [28], the second decay should not be fully reconstructed in order to get sufficient statistics in a relatively short time. However, for this second decay no vertexing technique can provide a σ_{z-z_r} smaller than $100\,\mu m$. From the point of view of the EPR test it is preferable to use the $t^* = t_1$ time only if it belongs to the fully reconstructed B meson. This is only satisfied on average in one half of the events and reduces the statistics by one half, but to a level still higher than achievable considering both fully reconstructed B mesons.

4.3 EPR test performed by mixing measurement method

A way to experimentally obtain the asymmetry (14), in order to test the quantum mechanical and local realistic predictions, (15) and (50) respectively, can be based on the measurement of the time-dependent mixing. Since $P[B^0(t_1); \overline{B^0}(t_r)] = P[\overline{B^0}(t_1); B^0(t_r)]$, $P[\overline{B^0}(t_1); \overline{B^0}(t_r)] = P[B^0(t_1); B^0(t_r)]$ hold both in QM and in LR, the asymmetry (14) can be written as

$$
\begin{aligned}
A(t_1, t_r) = \{&P[B^0(t_1); \overline{B^0}(t_r)] \\
&+ P[\overline{B^0}(t_1); B^0(t_r)] - P[\overline{B^0}(t_1); \overline{B^0}(t_r)] \\
&- P[B^0(t_1); B^0(t_r)]\} \\
/\{&P[B^0(t_1); \overline{B^0}(t_r)] + P[\overline{B^0}(t_1); B^0(t_r)] \\
&+ P[\overline{B^0}(t_1); \overline{B^0}(t_r)] + P[B^0(t_1); B^0(t_r)]\},
\end{aligned}
\tag{53}
$$

and also as

$$
\begin{aligned}
A(t_1, t_r) = \{&N(B^0, \overline{B^0})(t_1, t_r) - N(\overline{B^0}, \overline{B^0})(t_1, t_r) \\
&- N(B^0, B^0)(t_1, t_r)\} \\
/\ \{&N(B^0, \overline{B^0})(t_1, t_r) + N(\overline{B^0}, \overline{B^0})(t_1, t_r) \\
&+ N(B^0, B^0)(t_1, t_r)\},
\end{aligned}
\tag{54}
$$

where $N(B^0, \overline{B^0})$ represents the number of unlike-flavour events. In LR the asymmetry depends not only on Δt, as in QM, but also on t^*. However, the local realistic time dependence on Δt could be directly compared to that of QM, for instance by performing a partial time integration, namely an integration on t_1/τ for $t_1/\tau < 2$ with the constraint $\Delta t/\tau \leq 2$. Moreover, considering that a very large fraction of the events (double B decays) is included in the double requirement ($t_1/\tau < 3$ and $t_r/\tau < 3$), for a given $t_1/\tau < 3$ these events are characterized by values of Δt belonging to the time interval $[0, 3 - (t_1/\tau)]$, for which the local realistic prediction is always below the quantum mechanical one. Thus a total integration over t_1/τ is clearly allowed. This permits one to consider the following quantity with simplified time dependence:

$$
\begin{aligned}
&A(\Delta t) \\
&= \frac{N(B^0, \overline{B^0})(\Delta t) - N(\overline{B^0}, \overline{B^0})(\Delta t) - N(B^0, B^0)(\Delta t)}{N(B^0, \overline{B^0})(\Delta t) + N(\overline{B^0}, \overline{B^0})(\Delta t) + N(B^0, B^0)(\Delta t)}.
\end{aligned}
\tag{55}
$$

Therefore, considering a dilepton tagging approach to obtain experimentally the asymmetry (55), one can measure the asymmetry

$$
A(|\Delta t|) = \frac{N(l^+l^-) - N(l^+l^+) - N(l^-l^-)}{N(l^+l^-) + N(l^+l^+) + N(l^-l^-)},
\tag{56}
$$

by counting the number of like-sign and unlike-sign events, $N(l^\pm l^\pm)$ and $N(l^+l^-)$, as a function of $|\Delta t|$. As the Δz resolution for the cascade leptons is too large, in the mixing study the B meson flavour is correlated only with the sign of the direct leptons (distinguished by their harder momentum spectrum), and the cascade ones act as a source of mistagging [28]. To obtain the yield of dilepton events from semi-leptonic decays of $B^0\overline{B^0}$ pairs, several other backgrounds must be subtracted, such as dileptons from the non-resonant process at $\Upsilon(4S)$ (continuum) and hadrons misidentified as leptons (fake leptons).

4.4 Quantitative evaluation for test feasibility

Two methods, based on lifetime and mixing measurements, have been proposed to perform the EPR test. The former, needing a time-dependent measurement not only for Δt but also for t^*, requires an exclusive channel approach. Its need of larger statistics suggests the choice of the latter method to experimentally obtain the asymmetry (14) for an initial study. The BaBar relative accuracy of about 1% in Δm_B [28] with only one year data taking at nominal luminosity ($30\,fb^{-1}$) suggests that at the same time an accuracy on the asymmetry (14) largely sufficient to discriminate between QM and LR predictions should be obtained. The number of events required to obtain a given level of separation between the two theoretical predictions provides a quantitative evaluation of the power of the proposed test. The number of $B^0\overline{B^0}$ events to be produced, in order to measure an asymmetry A with a statistical error δA, is approximately given by

$$
N_{prod} \approx \frac{1}{(\delta A)^2} \cdot \frac{1}{D^2 \epsilon \mathrm{Br}},
\tag{57}
$$

where D represents the product of all dilution factors, ϵ is the total detection efficiency and Br ($\simeq 0.04$) is the branching ratio into the dileptonic final state. The corresponding needed luminosity integrated over one typical year is

$$
\mathcal{L} \equiv \int L \mathrm{d}t = \frac{N_{\mathrm{prod}}}{\sigma_{b\bar{b}}},
\tag{58}
$$

where $\sigma_{b\bar{b}} \simeq 1.2\,\mathrm{nb}$ is the cross section for $b\bar{b}$ production at the $\Upsilon(4S)$. Neglecting higher order effects due to the backgrounds, the total dilution factor D is simply given by the tagging dilution factor $d_{\mathrm{tag}} = (1 - 2\eta)$ where the mistag probability η is the probability that a $B^0\overline{B^0}$ pair is tagged as a B^0B^0 or $\overline{B^0}\overline{B^0}$ pair. Typical values are $\eta = 13\%$ and $\epsilon = 45\%$ [28]. N_{prod} has been evaluated requiring the separation between the integrated values of A_{QM} and A_{LR} to be at least 4σ over an appropriate interval of the decay time distribution for $B^0\overline{B^0}$ pairs. Considering that Δt

and t^* both greater than 0.3τ give a large difference between the two predictions, one obtains $\mathcal{L} \approx 1\,\text{fb}^{-1}$. This is enough to discriminate between the two theoretical predictions. The larger luminosity of $30\,\text{fb}^{-1}$, integrated in one nominal year, will allow a more detailed and stringent test to be made.

5 Conclusions

Developing a very general local realistic theory of correlated $B_d^0\overline{B_d^0}$, a non-Bell-type EPR test has been proposed to discriminate between quantum mechanics and local realism. Indeed the asymmetry for observing neutral B_d meson pairs with like and unlike flavour predicted by local realism is remarkably different from the quantum mechanical prediction. This test can be based on mixing and lifetime measurements to be carried out by the experiments at the asymmetric B factories, and therefore is not only experimentally feasible but can be effectively performed quite soon if the expectations on the collider luminosity and the detector performance will be fulfilled by the BaBar and Belle collaborations.

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